

DAN:

Sea  $x$  el ángulo entre dos vectores  $\mathbf{u}$  y  $\mathbf{v}$  en  $\mathbb{R}^2$  o  $\mathbb{R}^3$ . Entonces

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cdot \cos x$$

$$\cos x = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|}$$

PIDEN:

Demostrar que:

$$\bullet \quad \|v - u\|^2 = \|u\|^2 + \|v\|^2 \cdot \cos x$$

EJECUCION:

$$\begin{aligned} \|v - u\|^2 &= \|u\|^2 + \|v\|^2 \cdot \cos x \\ 2\|u\| \|v\| \cdot \cos x &= \|u\|^2 + \|v\|^2 - \|v - u\|^2 \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} - (\mathbf{v} - \mathbf{u}) \cdot (\mathbf{v} - \mathbf{u}) \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{u} \\ &= 2\mathbf{u} \cdot \mathbf{v} \\ \text{luego } \mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cos x \end{aligned}$$

BIBLIOGRAFIA:

Algebra lineal (octava edicion) - Bernard Kolman . David R. Hill